Operations Research Exam Topics, Autumn 2025

1. Breadth First Search (BFS) in undirected and directed graphs. Dijkstra's algorithm for finding cheapest paths for nonnegative cost functions. Finding cheapest paths in acyclic directed graphs. The Bellman-Ford algorithm for finding cheapest paths for consercative cost functions. Existence of feasible potential, Gallai's theorem.

2. Matchings in bipartite graphs. Augmenting path algorithm for finding a matching of maximum size. Kőnig's theorem on matchings and vertex covers. Egerváry's theorem on the maximum cost of a perfect matching. The Hungarian Method. The assignment problem.

3. Feasible flow, maximum flow, Max Flow Min Cut theorem. The Ford-Fulkerson algorithm for finding a maximum flow. The minimum cost integer flow problem. Algorithm for finding a minimum cost integer flow for every possible flow value. Applications: transportation problem, transshipment problem. Hoffman's theorem on the existence of feasible circulations.

4. Linear Programming (LP), Integer Programming (IP), Mixed Integer Programming (MIP). The set of feasible solutions of an LP is a polyhedron. Face, dimension, vertex. Characteristic cone, characteristic subspace. Active rows, subspace of directions, cone of directions at a point z. The smallest face containing z. Characterization of minimal faces of a polyhedron. Polyhedral cones, generated cones, Farkas Lemma.

5. Characterization of emptiness of a polyhedron. Decomposition into a polytope and a cone. Basic solutions, strong basic solutions. Existence of optimal strong basic solution for a linear program. Characterization of optimality. Primal and dual linear programs. Weak duality theorem and Strong duality theorem. Optimality conditions.

6. Totally unimodular matrices. Operations preserving total unimodularity, examples of TU matrices. Integrality of strong basic solutions. TU version of Farkas Lemma, TU version of strong duality theorem. Applications: Gallai's theorem, Hoffman's theorem, Egerváry's theorem. Feasible and infesible bases of a linear system $Ax = b, x \geq 0$. Simplex method for finding a feasible basis, or showing that there is no solution. Primal simplex method for optimization.