Polyhedral Combinatorics – Exam Topics, Spring 2026

1. Laminar and cross-free hypergraphs. Totally dual integral (TDI) systems. Edmonds-Giles teorem, Cook Theorem. Examples: Lucchesi-Younger theorem, matroids.

2. Perfect matching polyhedron, matching polyhedron, TDI-ness.

3. The separation problem for the matching polyhedron. Upper hull of T-joins. Clutter, blocker, covering polyhedron. Ideal clutters. Example: upper hull of T-cuts.

4. Anti-blocker, packing polyhedron. Conform and perfect clutters. Replication lemma, weak perfect graph theorem. Polyhedral characterization of cliques and stable sets in perfect graphs.

5. Bessy-Thomassé theorem.

6. The Colorful Carathéodory theorem (including the stronger version).

7. Polyhedra of sub- and supermodular functions. Discrete separation theorem. Greedy algorithm for base polyhedra.

8. Paramodular pairs, generalized polymatroids (g-polymatroids). Totally dual laminar (TDL) systems define g-polymatroids. Intersection of a g-polymatroid with a box or a plank.

9. G-polymatroids defined by intersecting and essentially intersecting paramodular pairs. Characterization of non-emptiness. Applications: forest polytope, 2-dimensional rigidity matroid. Schrijver's supermodular coloring theorem.

10. Base polyhedra defined by crossing submodular set functions, characterization of non-emptiness. Application: (k, ℓ)-arc-connected orientations. Submodular flows, characterization of existence when b is submodular. Application: (k, ℓ)-edge-connected orientation of mixed graphs, minimum cost orientation.